

# Elementary Particles and Spin Representations

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We emphasize that the group-theoretical considerations leading to  $SO(10)$  unification of electro-weak and strong matter field components naturally extend to space-time components, providing a truly unified description of all generation degrees of freedoms in terms of a single chiral spin representation of one of the groups  $SO(13,1)$ ,  $SO(9,5)$ ,  $SO(7,7)$  or  $SO(3,11)$ . The realization of these groups as higher dimensional space-time symmetries produces unification of all fundamental fermions in a single space-time spinor.

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Nowadays, it is largely accepted that at some intermediate energy scale elementary matter and non-gravitational fundamental interactions should go through an  $SO(10)$  grand unification [1, 2]. While it is open the question on what the detailed dynamics at that scale should be, we are very confident that the group theoretical framework is correct. Our belief mainly grounds on the fact that the complicated family structure reorganizes in a single (spin) representation. This simplicity is a primary attractive of the model. Unfortunately,  $SO(10)$  only offers a partial answer to the unification quest: 1. the family representation has to be supported by its complex conjugate to take into account antiparticles; 2. the couple of family/anti-family representations has to be replicated—at least three times—to take into account generations; 3. last but not the least, elementary fermions transform as composite objects under space-time and internal gauge transformations, a signal that gravity has not been taken into account. Over the years, many different ideas have been proposed to overcome this situation [3]. This paper presents an original viewpoint. We emphasize that the group-theoretical considerations leading to unification of electro-weak and strong matter field components naturally extend to space-time components, providing a truly unified description of all fundamental fermions in terms of a single chiral spin representation of a fourteen dimensional pseudo-orthogonal space-time group. Our consideration are mainly of cinematical nature, only providing a general framework in which the whole unification issue may possibly be reconsidered. We will only touch on the dynamical problem. In order to better illustrate our viewpoint, we start by focusing on the family structure and resume the key observations that lead to treat leptons and quarks on a similar footing.

Pati-Salam [4] idea of lepton number as a fourth color

is briefly summarized by recalling that the chiral spin representations<sup>1</sup>  $\mathbf{2}$  and  $\mathbf{2}'$  of  $SO(4) \simeq SU_L(2) \times SU_R(2)$  respectively transform as  $\mathbf{1} + \mathbf{1}$  and  $\mathbf{2}$  under the subgroup  $SU_L(2)$ , while the chiral spin representations  $\mathbf{4}$  and  $\bar{\mathbf{4}}$  of  $SO(6) \simeq SU(4)$  respectively transform as  $\mathbf{1} + \bar{\mathbf{3}}$  and  $\mathbf{1} + \mathbf{3}$  under the regularly embedded  $SU(3)$ . By inspecting product representations

$$\begin{aligned} (\mathbf{2}, \mathbf{4}) &= (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \bar{\mathbf{3}}) + (\mathbf{1}, \bar{\mathbf{3}}) \\ (\mathbf{2}, \bar{\mathbf{4}}) &= (\mathbf{2}, \mathbf{1}) + (\mathbf{2}, \mathbf{3}) \\ (\mathbf{2}', \mathbf{4}) &= (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}) \\ (\mathbf{2}', \bar{\mathbf{4}}) &= (\mathbf{2}, \mathbf{1}) + (\mathbf{2}, \bar{\mathbf{3}}) \end{aligned} \tag{1}$$

we realize that the complicated  $U(1) \times SU(2) \times SU(3)$  Standard Model matter representation appears much simpler in terms of  $SO(4) \times SO(6)$  (hypercharge also can be accommodated). The enlargement of the gauge group—without addition of matter freedoms—produces a remarkable simplification in the theory’s structure, allowing to treat leptons and quarks on the very same ground. Yet, the situation is not completely satisfactory. The elementary matter representation still is composite and the gauge group still has a direct product structure. The problem was overcome by Georgi [1], essentially by recalling that the chiral spin representations of the ten dimensional orthogonal group  $SO(10)$ —also the ones of the pseudo-orthogonal group  $SO(4, 6)$  do—transform as

$$\begin{aligned} \mathbf{16} &= (\mathbf{2}, \mathbf{4}) + (\mathbf{2}', \bar{\mathbf{4}}) \\ \bar{\mathbf{16}} &= (\mathbf{2}, \bar{\mathbf{4}}) + (\mathbf{2}', \mathbf{4}) \end{aligned} \tag{2}$$

under the regular subgroup  $SO(4) \times SO(6)$ . The further enlargement of the gauge group, still without addition of

<sup>1</sup>Throughout this paper we use the common notation of labelling representations by their dimension (in boldface). Negative chirality Weyl representations are distinguished by a prime ' or a bar  $\bar{\phantom{x}}$  when conjugate to the positive chirality ones.

matter freedoms, produces unification inside each family. All left-handed matter fields  $\nu_R^c$ ,  $(\nu, e)_L$ ,  $e_R^c$ ,  $u_R^c$ ,  $(u, d)_L$  and  $d_R^c$  appear as components of the single irreducible representation **16** while all right-handed matter  $\nu_R$ ,  $(\nu, e)_L^c$ ,  $e_R$ ,  $u_R$ ,  $(u, d)_L^c$  and  $d_R$  makes up its conjugate **16**. The gauge group is no longer a direct product. As far as electro-weak and strong interactions are concerned and regardless to the problems mentioned above,  $SO(10)$  grand unification appears so natural to seem unavoidable.

Besides indices labelling particles properties under internal gauge transformations, elementary matter fields carry space-time indices. In the framework discussed above, left-handed matter transforms according the chiral spin representation **2** of the space-time Lorentz group  $SO(3,1)$  and —independently— according the chiral spin representation **16** of the grand unified gauge group  $SO(10)$ ; right-handed matter transforms according **2** under space-time transformations and according **16** under internal gauge transformations. In this prospective the theory does not look much unified. Rather, its structure resembles the one of the  $SO(4) \times SO(6)$  model before  $SO(10)$  grand unification: the elementary matter representation is composite and the group of allowed transformations present the direct product structure  $SO(3,1) \times SO(10)$ . A possible objection is that, unlike in the Pati-Salam model, the two groups play different roles. The gauge group only acts on matter field components  $\delta\psi = \frac{1}{2}\epsilon_{ij}\Sigma^{ij}\psi$  not involving coordinate transformations. The space-time group involves both, field components and coordinates transformations. In the special relativistic context, in which GUTs are in general considered, the theory is assumed to be invariant under the combined Lorentz transformation  $\delta x^\mu = \epsilon^\mu{}_\nu x^\nu + \tau^\mu$ ,  $\delta\psi = \frac{1}{2}\epsilon_{\mu\nu}\Sigma^{\mu\nu}\psi$  with identical infinitesimal parameters  $\epsilon^{\mu\nu}$ . Taking into account gravity is equivalent to gauge the Lorentz group [5] by considering the generalized transformations in which the parameters  $\epsilon^{\mu\nu}$  and  $\tau^\mu$  become arbitrary functions of the coordinates. It is then possible to regard as independent  $\epsilon^{\mu\nu}$  and  $\xi^\mu \equiv \epsilon^\mu{}_\nu x^\nu + \tau^\mu$ . In this way, we can consider generalized transformations with  $\xi^\mu = 0$  but arbitrary  $\epsilon^{\mu\nu}$ . In the general relativistic context, coordinates and Lorentz field transformations become completely independent. The action of  $SO(3,1)$  on matter field components is absolutely analogue to the one of an internal gauge group.

It looks then natural to pursue a unified description of elementary matter by recalling that the left-handed chiral spin representation of the fourteen dimensional pseudo-orthogonal group transforms as

$$\mathbf{64}' = (\bar{\mathbf{2}}, \mathbf{16}) + (\mathbf{2}, \bar{\mathbf{16}}) \quad (3)$$

under the regular subgroup  $SO(3,1) \times SO(10)$ . For the

signature a few different choices are possible. We can add the 10 Euclidean directions to the 3 space-like ones or the to single time-like one. Moreover, since we have to deal with a non-compact group anyway, there is no longer reason to prefer  $SO(10)$  to  $SO(4,6)$  in unifying  $SO(4) \times SO(6)$ . In this case we also can add the 4 to the 3 and the 6 to the 1 or the 4 to the 1 and the 6 to the 3. In brief, the fourteen dimensional pseudo-orthogonal groups

$$SO(13,1), \quad SO(9,5), \quad SO(7,7), \quad SO(3,11) \quad (4)$$

present  $SO(3,1) \times SO(4) \times SO(6)$  as regular subgroup and correctly contain all generation degrees of freedoms —space-time and gauge, particles and antiparticles— in their self-(pseudo)conjugate left-handed chiral spin representation. A generation of elementary particles is described by a single field  $\psi_{\mathbf{64}'}(x)$  living in  $\mathbf{64}'$ .<sup>2</sup> The eventual enlargement of the gauge group produces a genuine unification inside each generation, still without addition of matter freedoms.

At least from the group-theoretical viewpoint, these considerations give a satisfactory solution to the first and third problem we mention in the introduction. The second problem, concerning families replication, apparently remains unsolved. Should we further extend the gauge group to include the missing degrees of freedoms? In order to answer this question we leave for a while the problem of elementary matter unification and turn our attention to fundamental interactions. The gauge groups of gravitational, hyper, weak and strong forces are regular subgroups of the fourteen dimensional pseudo-orthogonal groups (4) and their embedding is in some sense minimal. After symmetry breaking, four directions —identified with the physical space-time— transform under the Lorentz group; the remaining four+six —corresponding to an unphysical internal space— are respectively needed to realize  $SU(2) \subset SO(4)$  and  $SU(3) \subset SO(6)$  transformations (hypercharge is related to the relative rotation of weak and strong directions). From the point of view of fundamental interactions a further extension of the gauge group is unnecessary, if not undesirable. On the other hand, we still have to specify the sense in which the unified group has to be understood as gauge group. Given the pseudo-orthogonal structure and the fact that four of its defining directions form the physical space-time, the only reasonable choice seems that of accepting the remaining ten directions as more physical dimensions. We are lead to a fourteen dimensional space-time [6] with a pseudo-Euclidean geometry described by one of the groups (4). To make contact with low energy physics it is convenient to introduce local co-

<sup>2</sup>Some of these degrees of freedom have to be identified with ordinary spin.

ordinates  $x^\mu$ ,  $\mu = 0, 1, 2, 3$  and  $\xi^i$ ,  $i = 1, 2, \dots, 10$ , respectively parameterizing directions transforming under the subgroups  $SO(3, 1)$  and  $SO(4) \times SO(6)$ . The former are identified with ordinary space-time coordinates, the latter parameterize extra dimensions. Let us now go back to elementary matter and the families replication problem. At low energies a generation of elementary particles is described by a field  $\psi_{\mathbf{64}'}(x)$  only depending on ordinary space-time coordinates. In the higher dimensional model such a field simply is a left-handed space-time spinor, but —as any other higher dimensional field— this field depends on *all* the space-time coordinates, the ordinary  $x$  and the extra  $\xi$

$$\Psi_{\mathbf{64}'}(x, \xi) \quad (5)$$

For every given value of extra coordinates  $\psi_{\mathbf{64}', \xi}(x) \equiv \Psi_{\mathbf{64}'}(x, \xi)$  exactly contains the fields describing a generation of elementary particles. The whole left-handed space-time spinor includes infinite many copies of the generation structure. The realization of the unified group as space-time symmetry makes unnecessary any further enlargement of the gauge group. *All fundamental fermions are placed in a single representation of the space-time group.*

The extension of space-time offers a non-group-theoretical explanation of the existence of many copies of the generation structure, addressing a possible solution of the families replication problem. The crucial point remains that of understanding how many of these copies survive when space-time symmetry is broken and the number of dimensions is effectively reduced to four. The answer depends on the particular dynamical mechanism employed in breaking the symmetry and goes beyond the goal of this paper. We note however, that the needed symmetry breaking can occur in the fermion-fermion operator. The product of two left-handed spinors decomposes as

$$\mathbf{64}' \times \mathbf{64}' = \Lambda_1 + \Lambda_3 + \Lambda_5 + \Lambda_7^- \quad (6)$$

in terms of  $k$ -forms  $\Lambda_k$ , the 7-form being anti-self-dual. The associated curvature forms —respectively the closed 2-, 4-, 6- and 8-forms constructed by exterior differentiation— foliate space-time in 2-, 4- and 6-dimensional hyper-surfaces. This is exactly what we need to break the space-time group down to an effective  $SO(3, 1) \times U(2) \times U(3)$  through a Freund-Rubin [7] like mechanism.<sup>3</sup> In addition equation (6) indicates that chiral fourteen-dimensional spinors are massless. In brief, the 14-dimensional left-handed space-time spinor

<sup>3</sup>The 14-dimensional space-times group can be broken in different ways in direct products of (pseudo)orthogonal and (pseudo)unitary symmetries; (pseudo)unitary groups are obtained when the 2-form survives on 4- and 6-hyper-surfaces, providing a complex structure.

(5) might be the right object describing elementary matter. In addressing a dynamical model different options are open. A particularly intriguing one is to assume that  $\Psi_{\mathbf{64}'}$  is the only elementary field, while the higher dimensional metric structure —hence the low energy graviton and gauge bosons— arises as a collective excitation of fermion-fermion pairs [8, 9].

We conclude by considering the problem of discerning among the four possible choices of unified gauge group. A partial answer comes from the inspection of spin representations properties [10]. For signatures (13,1) and (9,5) the spin representations carry a quaternionic structure. On  $\mathbf{64}'$  it only is possible to define a pseudo-conjugation  $\tilde{C}$ ,  $\tilde{C}^2 = -1$ , commuting with the action of the group. For signatures (7,7) and (3,11) the spin representations carry a real structure instead. A genuine operation  $C$  of complex conjugation,  $C^2 = 1$ , is defined. It is compatible with the group action and commutes with the chiral operator. As a consequence the real and imaginary parts of  $\mathbf{64}'$  transform independently, forming irreducible real (Majorana) representations of the group. Particles and antiparticles live in a superposition of real and imaginary parts and the ones appear as the complex conjugate of the others. This is the common situation dealt with in field theory. The absence of a properly defined complex conjugation makes  $SO(13, 1)$  and  $SO(9, 5)$  not particularly attractive as unified groups —if not ruling them out. On the other hand, the spin representations of  $SO(7, 7)$  and  $SO(3, 11)$  are algebraically undistinguishable and a preference can not be expressed merely on this ground. From a purely aesthetical viewpoint, the total symmetry between components with opposite signature makes of  $SO(7, 7)$  a preferential candidate. In spite of technical problems, the idea of a space-time with an equal number of space and time dimensions, therefore treating space and time really on the same ground, appears particularly attractive.

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